Positivity and Fourier transforms of ${}_1F_2$ hypergeometric functions

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We consider the hypergeometric functions of type

$$ {}_1F_2\left(a; a+b, c; -\frac{x^2}{4}\right) \qquad (a>0, b>0, c>0, x\in\mathbb{R}) $$

and present or discuss the following results or aspects:

- Concerning the positivity, we revisit the work of Askey, Fields and Ismail, and Gasper and reprove some of their results by representing the hypergeometric functions as the integrals involving the squares of Bessel functions. We also improve some of their limiting cases and give a new proof for the positivity of Struve's functions.

- We show that the hypergeometric functions arise as band-limited signals by evaluating their Fourier transforms explicitly.

- As applications, we show how such a hypergeometric function may be used in constructing compactly supported reproducing kernels for Sobolev spaces.

Our method is based on a class of integral transforms defined by

$$ f \mapsto \int_0^\infty \Omega_\lambda(xt) f(t)\,dt \qquad (x\geq 0), $$

where the kernel Ω_λ stands for

$$ \Omega_\lambda(t) = \Gamma(\lambda+1)(t/2)^{-\lambda} J_\lambda(t) = {}_0F_1\left(\lambda+1;\, -t^2/4\right), $$

which will be called the Hankel-Schoenberg transform of f.